APPROXIMATE POWER OF TEST PROCEDURES BASED ON TWO PRELIMINARY TESTS IN THE MIXED MODEL

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Summary

Series formulae for power of the tests based on preliminary test(s) of significance have always been lengthy and tedious They take lot of time for evaluation even on third generation computers. In this paper, the authors have derived approximate formulae for power of three test procedures based on two preliminary test of significance in a three factor factorial experiment in a mixed model which give quite satisfactory results.

1. Discussion of the problem :

Consider a factorial experiment with three factors A, B and C at levels a, b and c respectively arranged in randomized block design with r blocks in which the effect A is fixed and effects B and C are random. The appropriate model is

$$X_{ijkl}=\mu+\alpha_i+\beta_j+\gamma_k+\rho_l+(\alpha\beta)_{ij}+(\alpha\gamma)_{ik}+(\beta\gamma)_{jk}+(\alpha\beta\gamma)_{ijk}+e_{ijkl} \quad ...(1.1)$$

To test the hypothesis, H_0 : $\alpha_i=0$ *vs.* H_1 : $\alpha_i>0$, about the effect A, the abridged ANOVA table is as follows:

From table 1 it is apparent that no interaction mean square is adequate to be taken as error mean square unless the interaction effect AB and/or AC are/is zero. Hence it becomes necessary to test first the existence of AB and/or AC by testing two hypotheses, namely, H_{01} : $\sigma^2_{AB}=0$ *vs.* $H\sigma^1_{01}>0$ and H_{02} : H_{02} : $\sigma^2_{AC}=0$ *vs.* H^1_{02} : $\sigma^2_{AC}>0$. The final test depends entirely upon

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Table 1

Analysis of variance

Source	*d.f.*	*M.S.*	*E.M.S.*
Effect A	$(a-1)=n_4$	V_4	$\sigma_e^2+r\sigma_{ABC}^2+rb\sigma_{AC}^2+rc\sigma_{AB}^2+rbc\sum_{i=1}^{a}\frac{\sigma_i^2}{a-1}=\sigma_4^2$
Doubtful Error₁			
AB	$(a-1)(b-1)=n_3$	V_3	$\sigma_e^2+r\sigma_{ABC}^2+rc\alpha_{AB}^2=\sigma_3^2$
Doubtful Error₂			
AC	$(a-1)(c-1)=n_2$	V_2	$\sigma_e^2+r\sigma_{ABC}^2+rb\sigma_{AC}^2=\sigma_2^2$
True Error ABC	$(a-1)(b-1)(c-1)=n_1$	V_1	$\sigma_e^2+r\sigma_{ABC}^2=\sigma_1^2$

the outcome of these tests of significance. Such tests are called test based on conditional specification. For a detailed study of such tests see Bancraft and Han [1]. In case, none of the first order interaction comes out to be zero, we have considered three Satterthwaite [6] type test statistics leading to three different test procedures, each consisting of four mutually exclusive steps, which are as follows :

Procedure I

$$\text{Step 1}: \frac{V_3}{V_1}\geqslant\beta_1,\ \frac{V_2}{V_1}\geqslant\beta_2,\ \frac{V_4+V_1}{V_3+V_2}\geqslant\beta_3$$

$$\text{Step 2}: \frac{V_3}{V_1}\geqslant\beta_1,\ \frac{V_2}{V_1}<\beta_2,\ \frac{V_4}{V_3}\geqslant\beta_4 \qquad \ldots(1.2)$$

$$\text{Step 3}: \frac{V_3}{V_1}<\beta_1,\ \frac{V_2}{V_{13}}\geqslant\beta_5,\ \frac{V_4}{V_2}\geqslant\beta_6$$

$$\text{Step 4}: \frac{V_3}{V_1}<\beta_1,\ \frac{V_2}{V_{13}}<\beta_5,\ \frac{V_4}{V_{123}}\geqslant\beta_7$$

Procedure II

$$\text{Step 1 : } \frac{V_3}{V_1} \geqslant \beta_1,\ \frac{V_2}{V_1} \geqslant \beta_2,\ \frac{V_4}{V_3+V_2-V_1} \geqslant \beta_{32} \qquad \text{...(1.3)}$$

Other three steps are same as in Procedure I.

Procedure III

$$\text{Step I : } \frac{V_3}{V_1} \geqslant \beta_1,\ \frac{V_2}{V_1} \geqslant \beta_2,\ \frac{V_4-V_3}{V_2-V_1} \geqslant \beta_{33} \qquad \text{...(1.4)}$$

Other three steps are same as in Procedure I.

where

$$\beta_1=F(n_3, n_1; \alpha_1),\ \beta_2=F(n_2, n_1; \alpha_2);\ \beta_3=F(\nu_1, \nu_2; \alpha_3)$$
$$\beta_4=F(\nu, n_3; \alpha_4),\ \beta_5=F(n_2, n_{13}; \alpha_5),\ \beta_6=F(\nu, n_2; \alpha_6)$$
$$\beta_7=F(\nu, n_{123}; \alpha_7),\ \beta_{32}=F(\nu, \nu_3; \alpha_3),\ \beta_{33}=F(\nu_4, \nu_5; \alpha_3)$$
$$n_{13}=n_1+n_3;\ n_{123}=n_2+n_{13}$$

and $V_{13}=(n_1V_1+n_3V_3)/n_{13};\ V_{123}=(n_1V_1+n_2V_2+n_3V_3)/n_{123}$

The mean square V_4 is distributed as non-central chi-square which is approximated to central chi-square using Patnaik's [5] approximation. According to this $n_4V_4/(c'\sigma_1^2)$ is distributed as central χ^2 with ν *d.f.* where ν is given by

$$\nu=n_4+\frac{4\lambda^2}{n_4+4\lambda};\ \lambda=\frac{n_4}{2}(\Theta_{14}^{-1}-1) \text{ and}$$

$$C'=1+\frac{2\lambda}{n_4+2\lambda} \qquad \text{.. (1.5)}$$

The degrees of freedom ν_m (=1, 2, 3, 4, 5) are obtained by the following formulae,

$$\nu_1=\left(\frac{\nu C'}{n_4}+1\right)^2\Big/\left(\frac{\nu C'^2}{n_4^2}+\frac{1}{n_1}\right)$$

$$\nu_2=(\Theta_{12}+\Theta_{13})^2\Big/\left(\frac{\Theta_{12}^2}{n_3}+\frac{\Theta_{13}^2}{n_2}\right) \qquad \text{...(1.6)}$$

$$\nu_3=(\Theta_{12}^{-1}+\Theta_{13}^{-1}-1)^2\Big/\left(\frac{1}{n_2\Theta_{12}^2}+\frac{1}{n_3\Theta_{13}^2}+\frac{1}{n_1}\right)$$

$$\nu_4=\left(\frac{\nu C'\Theta_{13}}{n_4}-1\right)^2\Big/\left(\frac{\nu C'^2\Theta_{13}^4}{n_4^2}+\frac{1}{n_3}\right)$$

$$\nu_5=(\Theta_{12}^{-1}-1)^2\Big/\left(\frac{1}{n_2\Theta_{12}^2}+\frac{1}{n_1}\right)$$

Also $\Theta_{1j}=\sigma_1^2/\sigma_j^2\ (j=2, 3, 4)$ which is never greater than unity.

The power of each of the three test procedures will be the sum of the probabilities of four mutually exclusive steps given under each procedure. These powers will lead to size whenever, H_0 is true. The condition for size of the test procedure I can be obtained by taking.

$$(\sigma_4^2+\sigma_1^2)/(\sigma_3^2+\sigma_2^2)=1$$

$$\text{or } \Theta_{13}^{-1}+\Theta_{12}^{-1}-\Theta_{14}^{-1}=1 \qquad \ldots(1.7)$$

It can easily be verified that the condition for size of the procedures II and III will come out to be the same as given by (1.7).

2. Approximate power of test procedure I

Let the probability of the four steps of (1.2) be denoted by P_i ($i=1, 2, 3, 4$) respectively. Derivation of the approximate power formulae is based on the following assumptions as suggested by Bozivich *et. al.* [2].

Letting n_1, n_2 and $n_3 \to \infty$ in such a way that n_2/n_1 and n_3/n_1 are finite. Hence V_i tends to σ_i^2 ($i=1, 2, 3$).

Thus, the probability of step 1 is

$$P_1=\left(\frac{V_3}{V_1}\geqslant\beta_1,\ \frac{V_2}{V_1}\geqslant\beta_2,\ \frac{V_4+V_1}{V_3+V_2}\geqslant\beta_3\right) \qquad \ldots(2.1)$$

Making use of the assumptions given above, we have $V_3/V_1\to\sigma_3^2/\sigma_1^2$, $V_2/V_1\to\sigma_2^2/\sigma_1^2$ and $(V_4+V_1)/(V_3+V_2)\to(V_4+\sigma_1^2)/\sigma_3^2+\sigma_2^2)$. Obviously, the three solitary test statistics are independent.

Therefore,

$$P_1 \doteq P\left(\frac{V_3}{V_1}\geqslant\beta_1\right)P\left(\frac{V_2}{V_1}\geqslant\beta_2\right)P\left(\frac{V_4+V_1}{V_3+V_2}\geqslant\beta_3\right) \qquad \ldots(2.2)$$

To evaluate the above probabilities we make use of the following standard relations :

$$P(F_{p,\,q}<F_o)=I\left(X;\ \frac{p}{2},\frac{q}{2}\right) \qquad \ldots(2.3)$$

$$P(F_{p,\,q}\geqslant F_0)=1-I\left(X;\ \frac{p}{2},\frac{q}{2}\right) \qquad \ldots(2.4)$$

where,

$X=\dfrac{pF_0}{q+pF_0}$ and $I\left(X;\ \dfrac{p}{2},\dfrac{q}{2}\right)$ is the normalised incomplete beta function. The probabilities given in (2.2) can be reduced to the form (2.4) in the following manner.

$$P_1 \doteq P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2} \geqslant \beta_1 \Theta_{13}\right) P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} \geqslant \beta_2 \Theta_{12}\right)$$

$$X\, P\left\{\frac{(V_4+V_1)/\left(\frac{C'\nu\sigma_1^2}{n_4}+\sigma_1^2\right)}{(V_3+V_2)/(\sigma_3^2+\sigma_2^2)} \geqslant \frac{(\Theta_{12}^{-1}+\Theta_{13}^{-1})\beta_3}{(C'\nu n_4^{-1}+1)}\right\}$$

or ...(2.5)

$$P_1 \doteq P\{F(n_3, n_1) \geqslant \beta_1 \Theta_{13}\}\, P\{F(n_2, n_1) \geqslant \beta_2 \Theta_{12}\}$$

$$X\, P\left\{F(\nu_1, \nu_2) \geqslant \left(\frac{(\Theta_{12}^{-1}+\Theta_{13}^{-1})\beta_3}{(C'\nu n_4^{-1}+1)}\right)\right\} \quad \text{...(2.5.1)}$$

$$\doteq \left\{1-I\left(X_1; \frac{n_3}{2}, \frac{n_1}{2}\right)\right\}\left\{1-I\left(X_2; \frac{n_2}{2}, \frac{n_1}{2}\right)\right\}$$

$$X\left\{1-I\left(X_3; \frac{\nu_1}{2}, \frac{\nu_2}{2}\right)\right\} \quad \text{...(2.5.2.)}$$

where

$$X_1 = \frac{n_3\beta_1\Theta_{13}}{n_1+n_3\beta_1\Theta_{13}}\,;\quad X_2 = \frac{n_2\beta_2\Theta_{12}}{n_1+n_2\beta_2\Theta_{12}}$$

and $$X_3 = \frac{\nu_1(\Theta_{12}^{-1}+\Theta_{13}^{-1})\beta_3}{(C'\nu n_4^{-1}+1)\,\nu_2+\nu_1(\Theta_{12}^{-1}+\Theta_{13}^{-1})\beta_3} \quad \text{...(2.6)}$$

Proceeding in the same manner and making similar assumptions as in case of P_1, we obtain the probabilities P_2, P_3 and P_4 for step 2, 3 and 4 respectively. Thus,

$$P_2 \doteq P\left(\frac{V_3}{V_1} \geqslant \beta_1\right) P\left(\frac{V_2}{V_1} < \beta_2\right) P\left(\frac{V_4}{V_3} \geqslant \beta_4\right) \quad \text{...(2.7)}$$

$$P_2 \doteq P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2} \geqslant \beta_1\Theta_{13}\right) P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} < \beta_2\Theta_{12}\right)$$

$$X\, P\left(\frac{n_4 V_4/C'\nu\sigma_1^2}{V_3/\sigma_3^2} \geqslant \frac{n_4\beta_4}{C'\nu\Theta_{13}}\right)$$

$$\doteq P\{F(n_3, n_1) \geqslant \beta_1 \Theta_{13}\}\; P\{F(n_2, n_1) < \beta_2\Theta_{12}\}$$

$$X\, P\left\{F(\nu, n_3) \geqslant \frac{n_4\beta_4}{\nu C'\Theta_{13}}\right\} \quad \text{...(2.7.1)}$$

Making use of the relations (2.3) and (2.4) we obtain

$$P_2 \doteq \left\{1-I\left(X_1; \frac{n_3}{2}, \frac{n_1}{2}\right)\right\} I\left(X_2; \frac{n_2}{2}, \frac{n_1}{2}\right) X$$

$$\left\{1-I\left(X_2;\ \frac{\nu}{2},\frac{n_3}{2}\right)\right\} \qquad \ldots(2.7.2)$$

In (2.7.2), X_1 and X_2 are same as given in (2.6)

and $\quad X_4=n_4\beta_4/(n_3\ C'\Theta_{13}+n_4\beta_4) \qquad \ldots(2.8)$

The probability of step 3 under the similar assumptions as discussed in case of P_1, is

$$P_3 \doteq P\left(\frac{V_3}{V_1}<\beta_1\right) P\left(\frac{V_2}{V_{13}}\geqslant\beta_5\right) P\left(\frac{V_4}{V_2}\geqslant\beta_6\right) \qquad (2.9)$$

$$\doteq P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2}<\beta_1\Theta_{13}\right) P\left(\frac{V_2/\sigma_2^2}{V_{13}/\sigma_1^2}\geqslant\beta_5\Theta_{12}\right)$$

$$x\ P\left(\frac{n_4V_4/\nu C'\sigma_1^2}{V_2/\sigma_2^2}\geqslant\frac{n_4\beta_6}{C'\nu\Theta_{12}}\right)$$

since the hypothesis $\sigma_1^2=\sigma_3^2$ is accepted.

or $\quad P_3 \doteq P\{F(n_3,n_1)<\beta_1\Theta_{13}\}\ P\{F(n_2,n_{13})\geqslant\beta_5\Theta_{12}\}$

$$x\ P\left\{F(\nu,n_2)\geqslant\frac{n_4\beta_6}{C'\nu\Theta_{12}}\right\} \qquad \ldots(2.9.1)$$

Applying the relations (2.3) and (2.4) we get

$$P_3 \doteq I\left(X_1;\ \frac{n_3}{2},\frac{n_1}{2}\right)\left\{1-I\left(X_5;\ \frac{n_2}{2},\frac{n_{13}}{2}\right)\right\}$$

$$\left\{1-I\left(X_6;\ \frac{\nu}{2},\frac{n_2}{2}\right)\right\} \qquad \ldots(2.9.2)$$

where $\quad X_5=n_2\beta_5\Theta_{12}/(n_{13}+n_2\beta_5\Theta_{12}) \qquad \ldots(2.10)$

and $\quad X_6=n_4\beta_6/(n_2C'\Theta_{12}+n_4\beta_6)$

Similarly the probability of step 4 is

$$P_4 \doteq P\left(\frac{V_3}{V_1}<\beta_1\right) P\left(\frac{V_2}{V_{13}}<\beta_5\right) P\left(\frac{V_4}{V_{123}}\geqslant\beta_7\right) \qquad \ldots(2.11)$$

$$\doteq P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2}<\beta_1\Theta_{13}\right) P\left(\frac{V_2/\sigma_2^2}{V_{13}/\sigma_1^2}<\beta_5\Theta_{12}\right)$$

$$x\ P\left(\frac{n_4V_4/C'\nu\sigma_1^2}{V_{123}/\sigma_1^2}\geqslant\frac{n_4\beta_7}{C'\nu}\right)$$

since the hypotheses $\sigma_1^2=\sigma_2^2$ and $\sigma_1^2=\sigma_3^2$ are accepted.

$$P_4 \doteq P\{F(n_3, n_1) < \beta_1 \Theta_{13}\}\, P\{F(n_2, n_{13}) < \beta_5 \Theta_{12}\}$$

$$\times P \left\{ F(\nu, n_{123}) \geqslant \frac{n_4}{C'\nu} \beta_7 \right\} \qquad \ldots(2.11.1)$$

with the help of the relations (2.3) and (2.4)

we get,

$$P_4 \doteq I\left(X_1;\ \frac{n_3}{2}, \frac{n_1}{2} \right) I \left(X_5;\ \frac{n_2}{2}, \frac{n_{13}}{2} \right)$$

$$\left\{ 1 - I \left(X_7;\ \frac{\nu}{2}, \frac{n_{123}}{2} \right) \right\} \qquad \ldots(2.11.2)$$

where X_1 and X_5 are as given earlier and $X_7 = n_2\beta_7/(n_{123}C' + n_4\beta_7)$ ...(2.12)

Thus the power of the test procedure I is the sum of the probabilities P_1, P_2, P_3 and P_4 given by (2.5.2), (2.7.2), (2.9.2) and (2.11.2) respectively.

3. APPROXIMATE POWER OF TEST PROCEDURE II

In test procedure II, only step 1 is different from that of procedure I. Hence we have to find the probability of step 1 and use the results for the last three steps from section 2.

Here again we make the assumptions as given in section 2 and on the basis of these assumptions it is easy to show that V_3/V_1, V_2/V_1 and $V_4/(V_3+V_2-V_1)$ are independent.

Thus the probability 'P_{21}' of step 1 of procedure II is

$$P_{21} \doteq \left(\frac{V_3}{V_1} \geqslant \beta_1 \right) P \left(\frac{V_2}{V_1} \geqslant \beta_2 \right) P \left(\frac{V_4}{V_3 + V_2 - V_1} \geqslant \beta_{32} \right) \ldots(3.1)$$

or

$$P_{21} \doteq P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2} \geqslant \beta_1\, \theta_{13} \right) P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} \geqslant \beta_2 \theta_{12} \right)$$

$$\times P\left\{ \frac{n_4 V_4 / C'\nu\sigma_1^2}{(V_3 + V_2 - V_1)/(\sigma_3^2 + \sigma_2^2 - \sigma_1^2)} \geqslant \frac{n_4(\sigma_3^2 + \sigma_2^2 - \sigma_1^2)\beta_2}{\nu C' \sigma_1^2} \right\}$$

$$\doteq P\{F(n_3, n_1) \geqslant \beta_1\theta_{13}\}\, P\{F(n_2, n_1) \geqslant \beta_2\theta_{12}\}$$

$$\times P \left\{ F(\nu, n_3) \geqslant \frac{n_4(\theta_{13}^{-1} + \theta_{12}^{-1} - 1)\beta_2}{C'\nu} \right\} \quad \ldots(3.1.1)$$

With the help of the relations (2.3) and (2.4) we get

$$P_{21} \doteq$$

$$\left\{ 1 - I\left(X_1; \frac{n_3}{2}, \frac{n_1}{2} \right) \right\} \left\{ 1 - I\left(X_2;\ \frac{n_2}{2}, \frac{n_1}{2} \right) \right\} \left\{ 1 - I\left(X_{32}; \frac{\nu}{2}, \frac{n_3}{2} \right) \right\}$$

...(3.1.2)

where X_1 and X_2 are same as given in (2.6) and

$$X_{32}=\frac{n_4(\theta_{13}^{-1}+\theta_{12}^{-1}-1)\beta_{32}}{C'\nu_3+n_4(\theta_{13}^{-1}+\theta_{12}^{-1}-1)\beta_{32}} \qquad \ldots(3.2)$$

Thus the power of the test procedure II will be the sum of the probabilities given by (2.7.2), (2.9.2), (2.11.2) and (3.1.2).

4. Approximate Power of Test Procedure III

Proceeding in the same manner, as in sections 2 and 3, the probability 'P_{3_1}' of step 1 of procedure III given in (1.4) is

$$P_{3_1}\doteq P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2}\geqslant\beta_1\theta_{13}\right)P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2}\geqslant\beta_2\theta_{12}\right)$$

$$\times P\left\{\frac{(V_4-V_3)/\left(\frac{\nu C'\sigma_1^2}{n_4}-\sigma_3^2\right)}{(V_2-V_1)/(\sigma_2^2-\sigma_1^2)}\geqslant\frac{(\sigma_2^2-\sigma_1^2)\beta_{33}}{\left(\frac{\nu C'\sigma_1^2}{n_4}-\sigma_3^2\right)}\right\} \qquad \ldots(4.1)$$

or

$$P_{3_1}\doteq P\{F(n_3,n_1)\geqslant\beta_1\theta_{13}\}\,P\{F(n_2,n_1)\geqslant\beta_2\theta_{12}\}$$

$$\times P\left\{F(\nu_4,\nu_5)\geqslant\left(\frac{1}{\theta_{12}}-1\right)\beta_{33}/\left(\frac{C'\nu}{n_4}-\frac{1}{\theta_{13}}\right)\right\} \qquad \ldots(4.1.1)$$

Making use of the relations (2.3) and (2.4) we get

$$P_{31}\doteq$$

$$\left\{1-I\left(X_1;\frac{n_3}{2},\frac{n_1}{2}\right)\right\}\left\{1-I\left(X_2;\frac{n_2}{2},\frac{n_1}{2}\right)\right\}\left\{1-I\left(X_{33};\frac{\nu_4}{2},\frac{\nu_5}{2}\right)\right\}$$

$$\ldots(4.1.2)$$

where X_1 and X_2 are same as given in (2.6)

and

$$X_{33}=\frac{\nu_4(\theta_{12}^{-1}-1)\beta_{33}}{\nu_5(C'\nu n_4^{-1}-\theta_{13}^{-1})+\nu_4(\theta_{12}^{-1}-1)\,\beta_{33}} \qquad \ldots(4.2)$$

Thus the power of the test procedure III is the sum of the probabilities given by (2.7.2), (2.9.2), (2.11.2) and (4.1.2).

5. Evaluation of Size and Power

Five sets of degree of freedom which have been taken for evaluation of size and power of the test procedures are :

Degrees of Freedom

	n_4	n_3	n_2	n_1
Set 1 :	4	16	8	32
Set 2 :	4	8	16	32
Set 3 :	2	6	4	12
Set 4 :	2	4	6	12
Set 5 :	2	4	4	8

The values of size and power for set 1 are summarised in the Appendix. The values for other sets are not given here for want of space. Since ν has been chosen as an even integral number, we have chosen its values as 2, 4, 6, ...and calculated corresponding values of θ_{14}. Values of θ_{12} and θ_{13} are chosen subject to the restrictions

$$\theta_{14} \leqslant \theta_{12} \text{ and } \theta_{14} \leqslant \theta_{13} \qquad \ldots(5.1)$$

$\nu_m (m=1, 2, 3, 4, 5)$ are generally in fractions. Values of F for fractional ν's are interpolated using Lavbscher's [4] interpolation formulas. Whenever the relation (1.7) among θ's holds, we get size of the test and otherwise 'the power'. For an experiment $n_i (i=1, 2, 3, 4)$ and θ's are fixed. Out of 7 α's, only $\alpha_p (p=1, 2, 5)$, the preliminary levels of significance are at the disposal of the experimenter. Hence a preliminary level is recommended so that the size of the test is such that the distortion is within the aproiri fixed tolerance limit, say, .10 and power is maximum. Table A.1 reveals that the size of the test procedures remains under control for $\alpha_p \geqslant .25$ and various values of θ_{ij}'s except for $\theta_{12}=\theta_{14}=.2928$ and for all α_p when $\theta_{ij}=1$.

In case of set 2 when $n_3=8$ and $n_2=16$, the size of the test procedure I and II remains under control for $\alpha_p \geqslant .50$ and for test procedure III for $\alpha_p \geqslant .25$. The size of the test for the remaining three sets having smaller values of n_1, n_2 and n_3 remains under control for the values of $\alpha_p \geqslant .50$ except when all θ's are unity. This increase in size may be due to the greater departure from our assumptions.

Table A.2 and other table (not given) for the power of the test procedures for the remaining four sets of degrees of freedom reveal that the power of the test procedures in general increases as the preliminary level decreases. Also the power of the test procedure I is greater than the other two test procedures. More over procedure III turns out to be the poorest with regard to the power. This result is in a agreement with the intuition of Davenport and Webster [3] that the approximate F-test using linear combinations of variances with positive coefficients perform better than those having negative coefficients. It may be pointed out that power for procedure *III* exists only for those values of ν_4 and ν_5 which are not less than unity.

Since the size of the test procedures is generally under control for $\alpha_p \geqslant .25$ and the power of the test procedures is adequately large for $\alpha_p = .25$, we recommend the use of preliminary tests at $\alpha_p = .25$.

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Appendix

TABLE A.1

Size of the test procedures

Set : $n_1=32$, $n_2=8$, $n_3=16$, $n_4=4$

θ_{12}	θ_{13}	θ_{14}	*Preliminary levels of significance*						
			1.0	*.50*	*.25*	*.10*	*.05*	*.01*	*0*
Procedure I									
1.0	1.0	1.0	.0500	.0500	.0500	.0500	.0500	,0500	.0500
1.0	.4226	.4226	.0500	.0572	.0728	.1014	.1256	.1783	.2601
1.0	.2928	.2928	.0500	.0530	.0658	.0972	.1351	.2259	.7595
1.0	.2254	.2254	.0500	.0476	.0500	.0622	.0873	.1580	.9137
1.0	.1835	.1835	.0500	.0503	.0517	.0572	.0657	.1061	.9738
.4226	1.0	.4226	.0500	.0724	.0983	.1510	.1456	.1973	.2601
.2928	1.0	.2928	.0500	.0689	.1053	.2179	.2173	.3598	.7595
.2254	1.0	.2254	.0500	.0604	,0825	.1943	.1625	.2856	.9137
.1835	1.0	.1835	.0500	.0558	.0689	.1786	.1208	.2116	.9738
Procedure II									
1.0	1.0	1.0	.0499	.0495	.0500	.0500	.0500	.0500	.0500
1.0	.4226	.4226	.0500	.0572	.0728	,1014	.1256	.1783	.2601
1.0	.2928	.2928	.0500	.0532	.0646	.0972	.1351	.2259	.7595
1.0	.2254	.2254	.0502	.0477	.0500	.0617	.0875	.1580	.9137
1.0	.1835	.1835	.0500	,0503	.0517	.0572	,0657	.1061	.9738
.4226	1.0	.4226	.0500	.0724	.0983	.1510	.1456	.1973	.2601
.2928	1.0	.2928	.0499	.0689	.1052	.2179	.2173	.3598	.7595
.2254	1.0	.2254	.0499	.0603	.0824	.1943	.1625	.2856	.9137
.1835	1.0	.1835	.0499	.0558	.0689	.1786	.1208	.2116	.9738
Procedure III									
.4226	1.0	.4226	.0466	.0709	.0976	.1510	.1454	.1973	.2601
.2928	1.0	.2928	.0496	.0688	.1502	.2179	.2173	.3598	.7495
.2254	1.0	.2254	.0496	.0602	.0824	.1943	.1625	.2856	.9137
.1835	1.0	.1835	.0497	.0557	.0688	.1786	.1208	.2116	.9738

TABLE A.2

Power of the Test Procedure

Set ; $n_1=32$, $n_2=8$, $n_3=16$, $n_4=4$

θ_{13}	θ_{14}	*Test proc.*	*Preliminary lerels of significance*						
			1.0	*.50*	*.25*	*.10*	*.05*	*.01*	*0*
1	*2*	*3*	4	*5*	*6*	*7*	*8*	*9*	*10*
$\theta_{12}=1.0$									
1.0	.4226		.2700	.2979	.2872	.2740	,2619	.2615	.2601
		II	.2032	.2812	.2830	.2740	.2586	.2615	.2601
.75	.2928	I	.3729	.4733	.5687	.6598	.6836	.7420	.7595
		II	.3065	.4487	.5605	.6598	.6725	.7419	.7595
.50	.2254	I	.3585	.3956	.4617	.5673	.6334	.7772	.9137
		II	,3223	.3784	.4543	.5673	.6158	.7771	.9137
.25	.1835	I	.1371	.1400	.1473	.1687	.1930	.2940	.9738
		II	.1332	.1381	.1463	.1687	.1894	.2939	.9738
$\theta_{12}=.75$									
1.0	.4226	I	.1737	.2195	.2385	.2533	.2440	.2573	.2601
		II	.1463	,2108	.2357	,2533	.2427	.2573	.2601
.75	.2928	I	.2677	.3666	.4763	.6058	.6252	.7260	.7595
		II	.2334	.3491	.4688	.6058	.6195	.7260	.7595
.50	.2254	I	.2764	.3293	.4069	.5398	.5686	.7640	.9137
		II	.2552	.3159	.3993	.5398	.5581	.7638	,9137
.25	.1835	1	.1099	.1206	.1339	.1666	,1651	.2905	.9738
		II	.1078	.1192	.1330	.1666	.1632	.2905	.9738

TABLE A.2—*Contd.*

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
θ_{12}=,50									
1,0	,4226	I	.0754	.1066	.1365	.1845	.1806	.2242	.2601
		II	,0727	.1054	.1361	.1845	.1804	.2242	.2601
		III	.0491	.0951	.1319	.1845	.1793	.2242	.2601
.75	.2928	I	.1354	.1959	.2854	.4552	.4584	.6303	,7595
		II	,1268	.1903	.2823	.4552	.4570	.6302	.7595
		III	.0704	.1541	.2626	.4552	.4475	.6297	.7595
.50	.2254	I	.1577	.1982	.2673	.4544	.4133	.6758	,9137
		II	.1506	,1925	.2633	.4544	.4099	.6754	.9137
		III	.0811	.1360	.2234	.4544	.3756	.6724	.9137
.25	.1835	I	.0689	.0790	.0948	.1588	.1147	.2633	.9738
		II	.0684	.0786	.0945	.1588	.1143	.2632	,9738
θ_{12}=.25									
1.0	.2254	I	.0660	.0799	.1087	.2278	.2069	.3480	.9137
		II	.0649	.0794	.1084	.2278	.2068	.3480	:9137
		III	.0629	.0686	.1079	.2278	.2067	.3483	.9137
.75	.2254	I	.0535	.0686	.0982	.2964	.1871	.3444	.9137
		II	.0533	,0685	.0981	.2964	.1871	.3444	.9137
		III	.0525	.0679	.0977	.2964	.1870	.3444	.9137
.50	.1835	I	.0659	.0786	.1057	.4383	.1646	.3623	.9738
		II	.0653	.0780	.1052	.4382	.1643	.3622	.9738
		III	.0631	.7060	.1035	.4380	.1632	.3618	.9738

Note : Power for procedure III does not appear in the table for the cases where ν_4 and/or ν_5 were either 0 or negative or less than unity.